A back-door brain drain

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ABSTRACT

The long run effect of migration solely by unskilled workers is that skilled workers in the home country acquire additional human capital yet their share in the country's workforce falls. Consequently, the country's average level of human capital is lowered.

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1. Introduction

There is an understandable, if not always justified concern that the migration of skilled workers from developing countries (the "brain drain") can harm these countries.[1] There is no such concern, however, over the migration of unskilled workers: if skilled workers are immune to the employment prospects that lure unskilled workers, the leakage of human capital is avoided.[2] The purpose of this paper is to investigate the possibility that this perception is misguided: good employment opportunities for unskilled work away from home affect human capital formation decisions at home. When the prospect of migration for attractive unskilled work abroad presents itself, the incentive to acquire skills is reduced, and skill formation is depressed.

To address this question, we construct a model in which the developing country produces a single consumption good, using a combination of skilled labor and unskilled labor. After fully characterizing the closed economy, we introduce the prospect of migration by incorporating a migration quota, set by the developed country. The quota is for unskilled workers; access to skilled occupations is reserved for natives (as has often been the case, for example, in the Gulf States). From the point of view of the effect on the supply of skills at home, this looks like the least threatening scenario.

We find that once unskilled workers can migrate, the average level of human capital increases. However, in the new equilibrium, the greater accumulation of human capital in response to the higher wage rate per unit of human capital is not sufficient to compensate for the fall in the share of skilled workers in the home country's workforce. Even though departures will not cause skill depletion, there will be skill erosion on account of reduced skill formation.

In Section 2 we construct a model of a developing economy closed to migration. In Section 3 we present the immediate and long-run realizations of the model once migration by unskilled workers is allowed. Section 4 contains a comparison between the open and closed settings. Section 5 concludes. The proofs of our Claims and Lemma are in the Appendix.

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[1] The Katz and Stark (1984) response to Kwok and Leland (1982) can serve as an early example of the polemic of the literature on the "brain drain."

[2] A notable exception is Stark et al. (2009), who show that in contrast to villages from which people migrate to the US to take up menial jobs, villages from which people migrate largely to Mexico City to take up skilled work in the formal and governmental sectors have a higher average level of schooling.

2. A closed economy equilibrium

Consider an economy without migration in which a large number of identical firms produce a single consumption good, the price of which is normalized at 1. This good is manufactured by a combination of skilled labor (human capital), which we denote by H, and unskilled (raw) labor, L. The economy is populated by N individuals (workers).

Each individual chooses a skill type. He can decide to remain unskilled, or to invest in skill acquisition. The decision to join the ranks of the skilled is accompanied by a concomitant choice of how much human capital to acquire. The wage of an unskilled worker is w_L, and the wage of a skilled worker is given by the wage rate per unit of human capital, w_H, times the number of units of human capital that the skilled worker chooses to acquire, θ. The twice differentiable convex cost function of acquiring human capital is $\theta^{\frac{K}{K-1}}$, where $K > 1$ is a constant.

The utility function of an individual of type $j \in \{H,L\}$ is

$$U_j = \ln(W_j), \tag{1}$$

where $W_j > 0$ is the individual's net wage earnings. Thus, for a skilled worker

$$W_H(\theta, w_H) = \theta w_H - \theta^{\frac{K}{K-1}}, \tag{2}$$

and for an unskilled worker

$$W_L(w_L) = w_L. \tag{3}$$

Given the wage payments, an individual who elects to become skilled chooses how much human capital to acquire by solving

$$\max_{\theta} W_H(\theta, w_H), \tag{4}$$

which, upon applying (2), yields as a unique solution $\theta^* = (\frac{K-1}{K} w_H)^{K-1}$; heretofore, an asterisk indicates an equilibrium value of a variable. The wellbeing of a skilled worker is then

$$W_H(w_H) = \frac{w_H \theta^*}{K}. \tag{5}$$

In equilibrium, both types of workers enjoy the same utility

$$\ln\left(\frac{w_H \theta^*}{K}\right) = \ln(w_L).$$

From here it follows that in the closed-economy equilibrium, the ratio of the wage of a skilled worker to the wage of an unskilled worker is

$$\frac{w_H \theta^*}{w_L} = K. \tag{6}$$

We assume that firms produce by means of a constant-returns-to-scale Cobb–Douglas technology

$$Y_i = (\theta N_{iH})^\alpha N_{iL}^{1-\alpha}, \tag{7}$$

where N_{iH}, N_{iL}, and θN_{iH} are the "quantities" of skilled labor, unskilled labor, and human capital, respectively, employed by firm i. Because each firm uses the same technology, we can just as well assume that there is only one firm in the market that employs the entire country's workforce and produces the consumption good according to the production function

$$Y = (\theta N_H)^\alpha N_L^{1-\alpha}. \tag{8}$$

This single firm chooses how many skilled and unskilled workers to employ in order to bring its profits to a maximum:

$$\max_{\{N_H, N_L\}} [(\theta N_H)^\alpha N_L^{1-\alpha} - w_H \theta N_H - w_L N_L],$$

subject to the constraint of the size of the working population

$$N_L + N_H \leq N. \tag{9}$$

Standard profit maximization requires equality of efficiency wages to the marginal product of each type of labor

$$w_H \theta = \frac{\alpha}{N_H} Y \tag{10}$$

$$w_L = \frac{1-\alpha}{N_L} Y. \tag{11}$$

From dividing (10) by (11) and rearranging, we derive the relative demand for labor,

$$\frac{N_H}{N_L} = \frac{\alpha}{1-\alpha} \frac{w_L}{w_H \theta}, \tag{12}$$

and from (6) and (12) we get that

$$\frac{N_H}{N_L} = \frac{\alpha}{K(1-\alpha)}. \tag{13}$$

In conjunction with the constraint of the size of the working population (9), Eqs. (10), (11), and (13) yield the endogenous variables of the market equilibrium:

$$\theta^* = [\alpha^\alpha (1-\alpha)^{1-\alpha} (K-1) K^{-\alpha}]^{\frac{K-1}{K(1-\alpha)+\alpha}} \tag{14}$$

$$w_H^* = [\alpha^\alpha (1-\alpha)^{1-\alpha} [K^K (K-1)^{-(K-1)}]^{1-\alpha}]^{\frac{1}{K(1-\alpha)+\alpha}} \tag{15a}$$

$$w_L^* = [\alpha^{K\alpha} (1-\alpha)^{K(1-\alpha)} [K^{-K} (K-1)^{K-1}]^\alpha]^{\frac{1}{K(1-\alpha)+\alpha}} \tag{15b}$$

$$N_H^* = \frac{\alpha}{K(1-\alpha)+\alpha} N \tag{16a}$$

$$N_L^* = \frac{K(1-\alpha)}{K(1-\alpha)+\alpha} N. \tag{16b}$$

3. An economy open to migration

3.1. The general setting

Consider now the possibility of migration to a foreign economy, F, that is more developed than the home (developing) country, D. The labor market of F is characterized by wages $w_L^f > w_L^*$, $w_H^f > w_H^*$, and $w_L^f < w_H^* \theta^*$, where w_L^f and w_H^f – the foreign country's wages for unskilled workers and for skilled workers, respectively – are given and are exogenous to the model. We assume that w_L^f differs enough from w_L^* and $w_H^* \theta^*$ for the impact of migration on wages not to interfere with the ordering $\tilde{w}_L < w_L^f < \tilde{w}_H \tilde{\theta}$, where we use a tilde to indicate the value of a variable in the open-to-migration setting. Consistent, for example, with the migration policy of the Gulf States at some times, we assume that F confines access to its highly-paid skilled jobs to natives, and allows migrants in to work only in unskilled jobs.

Let M denote the number of migrants. We assume that F admits migrants up to quota $\bar{M} < N_L^*$, and therefore, the constraint $M \leq \bar{M}$ binds with equality. Let those who end up as migrants be selected randomly from the pool of unskilled workers such that each unskilled worker migrates to and works in F with probability $p \in (0,1)$, or stays and works in D with the complementary probability $1 - p$.[3] Thus,

$$p \equiv \frac{\bar{M}}{N - \tilde{N}_H}.$$

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[3] Stark et al. (1997, 1998) introduced this protocol.

Assuming that $\bar{M}$ is sufficiently small in comparison with the number of unskilled workers in D, each individual's decision (whether to remain unskilled or join the ranks of skilled) has only a marginal effect on the total number of skilled workers and consequently, on the probability of migration. Thus, individuals perceive p as a parameter, and in what follows we employ the notation p rather than $p(\tilde{N}_H)$, unless the context mandates otherwise.

We also assume that the number of births remains unchanged when the economy opens up to migration. This assumption is justified when migration is temporary and/or when migrants are not accompanied by their families, and again corresponds to a feature of migration from Asia to the Gulf States (Lucas, 2005).

As before, each individual decides whether or not to acquire skills, bearing in mind his migration prospects. After acting on his decision, the individual has a one-shot chance to migrate.

3.2. The "immediate" run

Claim 1. *In the "immediate" run, upon the migration of $\bar{M}$ unskilled workers, the following changes take place:*

(a) *The average level of human capital in the home country increases.*
(b) *The wages of skilled (unskilled) workers decrease (increase).*

When employment prospects abroad present themselves, the returns to being unskilled rise; new arrivals on the labor market prefer to remain unskilled rather than to engage in human capital formation. This is due to both the employment prospects abroad and to higher (lower) domestic wages for unskilled (skilled) work. The economy undergoes a transition where the new unskilled workers replace the oldest, both skilled and unskilled workers, and consequently lower (increase) the marginal product and wages of unskilled (skilled) work. This process continues until the new individuals find human capital acquisition to be as rewarding as remaining unskilled; an open-to-migration equilibrium is reached.

3.3. The market equilibrium

In calculating the open-to-migration equilibrium, the steps in Section 2.2 are repeated with two modifications:

(a) The size of the working population constraint (9) now accounts for migration, namely $\tilde{N}_H + \tilde{N}_L \leq N - \bar{M}$.
(b) The requirement that utility subject to (11) is equal to utility subject to (3), namely that $\ln(w_H \theta^*/K) = \ln(w_L)$, is now replaced by the requirement of utility subject to (11) being equal to the *expected* utility

$$p \ln(W_M) + (1-p) \ln(\tilde{W}_L), \tag{17}$$

subject to (3) and to $W_M = w_L^f$, namely that

$$\ln(\tilde{w}_H \tilde{\theta}^*/K) = p \ln(w_L^f) + (1-p) \ln(\tilde{w}_L).$$

Following the steps taken in Section 2.2 and incorporating (a) and (b) enables us to derive the following set of equations where, to ease the presentation, we define $p^* \equiv \frac{\bar{M}}{N - \tilde{N}_H^*}$, $k \equiv (\frac{w_L^*}{w_L^f})^{\frac{p^*}{K(1-\alpha)+(1-p^*)\alpha}}$, and $l \equiv k^{K(1-\alpha)+\alpha}$:

$$\tilde{\theta}^* = \theta^* k^{-(K-1)(1-\alpha)} \tag{18}$$

$$\tilde{w}_H^* = w_H^* k^{-(1-\alpha)} \tag{19a}$$

$$\tilde{w}_L^* = w_L^* k^{\alpha} \tag{19b}$$

$$\tilde{N}_H^* = \frac{\alpha l}{K(1-\alpha) + \alpha l}(N - \bar{M}) \tag{20a}$$

$$\tilde{N}_L^* = \frac{K(1-\alpha)}{K(1-\alpha) + \alpha l}(N - \bar{M}). \tag{20b}$$

We next formulate the following lemma.

Lemma 1. $\tilde{\theta}^*$, $\tilde{w}_H^*$, $\tilde{w}_L^*$, $\tilde{N}_H^*$, and $\tilde{N}_L^*$ exist, and are unique.

The Lemma implies that $0 < \tilde{N}_H^* < N - \bar{M}$ and thus that $0 < p^* < 1$, $0 < k < 1$, and $0 < l < 1$.

4. Opening up to migration from the home country: repercussions for human capital formation, wages, and skill distribution

Claim 2. *Once unskilled workers can migrate, skilled workers choose to form more human capital than when no such migration is possible.*

Claim 3. *Once unskilled workers can migrate, the wages of skilled workers, both per unit of human capital and per worker, are higher, and the wages of unskilled workers who stay behind in the home country are lower than when migration is not possible.*

Claim 3 reveals that the (expected) gain from an opening to migration that is confined to unskilled workers affects skilled workers who end up strictly better off than they were in the no-migration setting. The higher wage rate per unit of human capital induces skilled workers to become more skilled than in the absence of migration by unskilled workers. This result is surprising in the sense that on the basis of our related work on the beneficial repercussions of a brain drain, we might have predicted a lower accumulation of human capital by skilled workers as a response to the tilting of the expected returns to skill in favor of the no-skill option.[4]

Claim 4. *An economy from which unskilled workers can migrate produces more unskilled and fewer skilled workers than an economy closed to such migration.*

Claim 5. *The combination of enhanced human capital formation, reduced numbers of skilled workers, and increased numbers of unskilled workers results in a fall in the average level of human capital in the home country.*

The average level of human capital is lower than in the closed economy because, in the open economy setting, skilled workers are costlier to the firms than they are in the closed economy setting (cf. Claim 3). This reduces the demand for (and thereby the production of) skilled labor in the new equilibrium. Consequently, not only does the total "stock" of human capital go down, but so does the average level of human capital in the economy.

5. Conclusions

In a model in which, prior to entering the labor market, an individual decides whether to remain unskilled or (and, if so, to what extent) to acquire human capital in order to become a skilled worker, we have shown that the possibility of migrating to a developed country for some people changes the incentive structure for others and, consequently, the equilibrium outcome of the economy. The revision of wages that makes unskilled labor cheaper means that in the new equilibrium the home country's firms employ fewer skilled workers in relation to unskilled workers than they do in the closed economy setting. Simultaneously, a higher wage rate per unit of human capital induces skilled workers

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[4] Cf. Stark et al. (1997, 1998), Stark and Wang (2002), and Fan and Stark (2007).

to acquire more human capital than before. While not experiencing a "brain drain," the home country suffers from "brain depletion," manifested as a decline in its average level of human capital; the enhanced quality of skilled workers proves to be insufficient to compensate for the decline in their quantity.

If the sharp dichotomy in the prospects of migration by skill level assumed in the model does not match up to the often less than clear reality, then, as long as the migration prospects are not skill neutral and are biased in favor of unskilled work, the qualitative results if not the precise predictions of our model will continue to hold.

Appendix

Proof of Claim 1. (a) Because $\tilde{N}_H = N_H^*$, $\tilde{\theta} = \theta^*$, and $\bar{M} > 0$, we have that $\frac{\tilde{\theta}\tilde{N}_H}{N - \bar{M}} = \frac{\theta^* N_H^*}{N - \bar{M}} > \frac{\theta^* N_H^*}{N}$.

(b) Factors of production are paid their respective marginal products, which are $w_H = \alpha(\frac{N_L}{\theta N_H})^{1-\alpha}$ for skilled work, and $w_L = (1 - \alpha)(\frac{\theta N_H}{N_L})^{\alpha}$ for unskilled work. Because $\tilde{N}_L = N_L^* - \bar{M}$, $\tilde{N}_H = N_H^*$, $\tilde{\theta} = \theta^*$, and $\bar{M} > 0$, we have that $\tilde{w}_H = \alpha(\frac{N_L^* - \bar{M}}{\theta^* N_H^*})^{1-\alpha}$ $< \alpha(\frac{N_L^*}{\theta^* N_H^*})^{1-\alpha} = w_H^*$, and that $\tilde{w}_L = (1 - \alpha)(\frac{\theta^* N_H^*}{N_L^* - \bar{M}})^{\alpha}$ $> (1 - \alpha)(\frac{\theta^* N_H^*}{N_L^*})^{\alpha} = w_L^*$. $\square$

Proof of Lemma 1. We define $f(\tilde{N}_H) \equiv \tilde{N}_H$, and $g(\tilde{N}_H) \equiv \frac{\alpha l(p(\tilde{N}_H))}{K(1-\alpha)+\alpha l(p(\tilde{N}_H))}(N - \bar{M})$. We have that $\frac{\partial f}{\partial \tilde{N}_H} > 0$, and that $\frac{\partial g}{\partial \tilde{N}_H} < 0$. For $\tilde{N}_H = 0$, we have that $f(\tilde{N}_H) < g(\tilde{N}_H)$, whereas for $\tilde{N}_H = N - \bar{M}$, we have that $f(\tilde{N}_H) > g(\tilde{N}_H)$. Therefore, because $\tilde{N}_H \in (0, N - \bar{M})$, there is exactly one value of $\tilde{N}_H$ in its domain for which $f(\tilde{N}_H) = g(\tilde{N}_H)$. We denote this value by $\tilde{N}_H^*$. When used in (18) through (20b), $\tilde{N}_H^*$ determines the other endogenous variables as well. $\square$

Proof of Claim 2. Because $k < 1$, it follows from (18) that $\frac{\tilde{\theta}^*}{\theta^*} = k^{-(K-1)(1-\alpha)} > 1$. $\square$

Proof of Claim 3. Because $k < 1$, it follows from (19a) that $\frac{\tilde{w}_H^*}{w_H^*} = k^{-(1-\alpha)} > 1$, and from (19b) that $\frac{\tilde{w}_L^*}{w_L^*} = k^{\alpha} < 1$. Because from Claim 2 $\frac{\tilde{\theta}^*}{\theta^*} > 1$, and $\frac{\tilde{w}_H^*}{w_H^*} > 1$, that $\frac{\tilde{w}_H^*\tilde{\theta}^*}{w_H^*\theta^*} > 1$ holds as well. $\square$

Proof of Claim 4. Because $l < 1$, it follows from (16a) and (20a) that $\frac{\tilde{N}_H^*}{N_H^*} = \frac{[K(1-\alpha)+\alpha]l}{K(1-\alpha)+\alpha l}\frac{N-\bar{M}}{N} < 1$, and from (16b) and (20b) that $\frac{\tilde{N}_L^*+\bar{M}}{N_L^*} = \frac{K(1-\alpha)+\alpha}{K(1-\alpha)+\alpha l}(1 + \frac{\alpha l}{K(1-\alpha)}\frac{\bar{M}}{N}) > 1$. $\square$

Proof of Claim 5. In the no-migration equilibrium, the number of skilled workers and the size of the workforce are given,

respectively, by (16a) and N. Thus, the share of skilled workers in the total workforce is

$$\frac{N_H^*}{N} = \frac{\alpha}{K(1-\alpha)+\alpha}. \tag{A.1}$$

In the open-to-migration setting, the number of skilled workers and the size of the workforce are given, respectively, by (20a) and $N - \bar{M}$. Consequently, the share of skilled workers in the economy is

$$\frac{\tilde{N}_H^*}{N - \bar{M}} = \frac{\alpha l}{K(1-\alpha)+\alpha l}. \tag{A.2}$$

For Claim 5 to hold, the following inequality has to obtain

$$\frac{\tilde{N}_H^*}{N - \bar{M}}\tilde{\theta}^* - \frac{N_H^*}{N}\theta^*$$

$$= \frac{\alpha l}{K(1-\alpha)+\alpha l}\theta^* k^{-(K-1)(1-\alpha)} - \frac{\alpha}{K(1-\alpha)+\alpha}\theta^* < 0. \tag{A.3}$$

Upon rearranging, (A.3) can be simplified to

$$[K(1-\alpha)+\alpha]k < K(1-\alpha)+\alpha k^{K(1-\alpha)+\alpha}. \tag{A.4}$$

Drawing on Bernoulli's inequality,[5] because $0 < k < 1$ and because $K(1-\alpha)+\alpha > 1$, we get that

$$1 + (k-1)[K(1-\alpha)+\alpha] < [1 + (k-1)]^{K(1-\alpha)+\alpha}. \tag{A.5}$$

We can now substitute the left hand side of (A.5) for $k^{K(1-\alpha)+\alpha}$ in the right hand side of (A.4) to get that (A.4) holds as long as the following inequality holds:

$$k < 1 + (k-1)\alpha,$$

which then simplifies to the condition $k < 1$. $\square$

References

Fan, C.S., Stark, O., 2007. International migration and "educated unemployment." Journal of Development Economics 83 (1), 76–87.

Katz, E., Stark, O., 1984. Migration and asymmetric information: comment. American Economic Review 74 (3), 533–534.

Kwok, V., Leland, H., 1982. An economic model of the brain drain. American Economic Review 72 (1), 91–100.

Lucas, R.E.B., 2005. International Migration and Economic Development: Lessons from Low-Income Countries. Edward Elgar, Cheltenham.

Stark, O., Boucher, S., Taylor, J.E., 2009. A gain with a drain? Evidence from rural Mexico on the new economics of the brain drain. In: Kornai, J., Mátyás, L., Roland, G. (Eds.), Corruption, Development, and Institutional Design. Palgrave Macmillan, Basingstoke.

Stark, O., Helmenstein, C., Prskawetz, A., 1997. A brain gain with a brain drain. Economics Letters 55 (2), 227–234.

Stark, O., Helmenstein, C., Prskawetz, A., 1998. Human capital depletion, human capital formation, and migration: a blessing or a "curse"? Economics Letters 60 (3), 363–367.

Stark, O., Wang, Y., 2002. Inducing human capital formation: migration as a substitute for subsidies. Journal of Public Economics 86 (1), 29–46.

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[5] For any $x > -1$, $(1+x)^r > 1 + xr$ for $r > 1$, and $(1+x)^r < 1 + xr$ for $0 < r < 1$.